Ramanujan Congruences for Infinite Family of Partition Functions

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The partition function $p_{[1^c\ell^d]}(n)$ can be defined using the generating function,

$$\sum_{n=0}^{\infty} p_{[1^c\ell^d]}(n)q^n = \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^c(1-q^{\ell n})^d}.$$

In this paper, we prove infinite families of congruences for the partition function $p_{[1^c\ell^d]}(n)$ modulo powers of ℓ (where ℓ is a prime ≤ 13) for any integers c and d (with $c > 0$ and $d \geq -2$ for $\ell \neq 11$). We use Hecke operators, explicit basis of the vector space of modular functions of the congruence subgroup $\Gamma_0(\ell)$ and work of Atkin and Gordon on proving congruences for the partition function $p_{-k}(n)$.